

The papers, though varying in tone and flavor, are all concerned with the problems one encounters when trying to do calculations in topology or homological algebra. Usually in mathematics, and especially in topology, the interesting canonical invariants that one wishes to determine are given in terms of certain obstruction groups like cohomology or homotopy groups. These groups are in turn defined to be the quotient of some very infinite object by another. The actual computation of these invariants poses therefore new and, as it turns out, interesting problems.

It is, for a start, shown in the first paper, by D. J. Anick, the only paper with a genuine computer science flavor, that the subject matter is difficult: he shows that computing the homotopy groups  $\pi_n(X) \otimes \mathbf{Q}$  of finite simply connected CW-complexes  $X$  is at least as hard as any problem that can be solved non-deterministically in polynomial time. The difficulties one encounters, in theory and in practice, are vividly illustrated by three papers that are concerned with calculations pertaining to the higher homotopy groups of the spheres, and by several other papers concerned with explicit computations: regarding knots, immersions of  $\mathbf{P}^n(\mathbf{R})$  in Euclidean spaces, bifurcation theory and the classifying space of the generalized dihedral group of order 16.

The volume contains furthermore some papers on general-purpose algorithms to compute cohomology groups, syzygies, etc. and discussions of computer algebra packages suitable for "topological calculations". Finally, there are two papers by Milnor and by Handler, Kauffman and Sandin on the Mandelbrojt set.

R. S.

**21[11A15, 11Y55].**—SAMUEL S. WAGSTAFF, JR., *Table of all Carmichael numbers  $< 25 \cdot 10^9$* , 38 computer output sheets deposited in the UMT file.

This table of the 2163 Carmichael numbers  $< 25 \cdot 10^9$  was placed in the UMT file in connection with the paper [2]. It has not been reviewed until now for reasons too complicated to set down here. It is reviewed now because of Jaeschke's UMT table described in the following review.

Wagstaff's table has 14 columns. Col. 1 has the sequential number of the Carmichael number (CN) from 1 to 2163. Col. 2 is the CN from CN = 561 to CN = 24991309729. Cols. 3–12 give a characterization of the CN for each of the first ten prime bases:  $p = 2$  through  $p = 29$ . If the CN is not a *strong pseudoprime* to the base  $p$  (and this is usually the case), the respective column states "weak". See, e.g., Definition 44 in [3, p. 227]. If  $p$  divides CN, the column is left blank. If

$$\text{CN} = t \cdot 2^s + 1,$$

with  $t$  odd, and if

$$(*) \quad p^t \equiv 1 \pmod{\text{CN}},$$

the column states “ST + 1”, while if CN is a strong pseudoprime to the base  $p$  but (\*) does not hold, the column states “ST – 1”.

Col. 13 lists the number (3 through 7) of prime factors in CN, and Col. 14 gives the factorization of CN. The reviewer easily checked that the three Carmichaels  $< 25 \cdot 10^9$  that are “acceptable Perrin composites” [1] are in the table as CN #1353, #1375 and #2142. But the fourth Carmichael that is an acceptable Perrin composite is beyond this table, since it equals  $43234580143 = 223 \cdot 5107 \cdot 37963$ . See the next review.

D. S.

1. G. C. Kurtz, Daniel Shanks, and H. C. Williams, *Fast primality tests for numbers less than  $50 \cdot 10^9$* , Math. Comp. **46** (1986), 691–701.
2. Carl Pomerance, J. L. Selfridge and Samuel S. Wagstaff, Jr., *The pseudoprimes to  $25 \cdot 10^9$* , Math. Comp. **35** (1980), 1003–1026.
3. Daniel Shanks, *Solved and unsolved problems in number theory*, 3rd ed., Chelsea, New York, 1985.

**22[11A15, 11Y55].**—GERHARD JAESCHKE, *Table of all Carmichael numbers  $< 10^{12}$* , 21 computer output sheets deposited in the UMT file.

This table of the 8238 Carmichael numbers (CN)  $< 10^{12}$  was placed in the UMT file in connection with the paper [1]. They are listed 395 per page in five columns and 79 rows. No other information is given; compare the elaborate detail in the previous review. Thus, even to determine that  $43234580143$ , which is mentioned in the previous review, is CN #2652, requires a moderate effort. The present table, therefore, supersedes Wagstaff’s table only in part.

Three points about [1] may be mentioned here. A CN may be defined as a number that satisfies

$$a^{\text{CN}} \equiv a \pmod{\text{CN}}$$

for all integers  $a$ . This is both simpler and more general than the definition given in [1]. Even a casual glance at the table shows that most (?) of the CN end in the decimal digit 1. This has long been known. In [1], the CN are analyzed (mod 12) but not (mod 10). Swift’s earlier UMT table of the 646 CN  $< 10^9$  has an “Author’s summary” [2] wherein CN that are products of three primes are also analyzed.

As submitted, each page of the present table had a two-inch solid black band at the top of the page. After determining that there was no information here, the reviewer boldly sliced off this top with a paper trimmer. This (a) reduced the space requirement of the table in the UMT file and (b) enabled the reviewer to appropriately celebrate the 200th anniversary of the French Revolution.

D. S.

1. Gerhard Jaeschke, *The Carmichael numbers to  $10^{12}$* , Math. Comp. **55** (1990), 361–367.
2. J. D. Swift, *Table of Carmichael numbers to  $10^9$* , Review **13**, Math. Comp. **29** (1975), 338–339.